10CS34

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Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. For any three sets A, B and C, prove that following $A \cap (B \Delta C) = (A \cap B) \Delta C = (A \cap B) \Delta (A \cap C)$. **(06 Marks)**

b. In a survey of 60 people it was found that 25 read weekly magazines , 26 read fortnightly magazines , 26 read monthly magazines , 9 read both weekly and monthly magazines , 11 read both weekly and fort nightly magazines , 8 read both fortnightly and monthly magazines and 3 read all three magazines. find
i) the number of people who read at least one of the three magazines and
ii) the number of people who read exactly one magazine. **(06 Marks)**

c. A certain soccer team wins (w) with probability 0.6, losses (L) with probability 0.3, and ties (T) with probability 0.1. The team plays three games over the weekend :
i) Determine the elements of the event A that the team wins at least twice and does not lose ; Also fine P(A).
ii) Determine the elements of the event B that the team wins, losses and ties in the same order ; Also find P(B). **(08 Marks)**

2 a. Prove the following :
i) $P \to (q \to r) \Leftrightarrow (P \wedge q) \to r$ ii) $[\neg P \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (P \wedge r) \Leftrightarrow r$. **(06 Marks)**

b. Express the following propositions in terms of only Nand and only Nor connectives :
i) $\neg P$ ii) $P \wedge q$ iii) $P \vee q$ iv) $P \to q$ v) $P \leftrightarrow q$. **(06 Marks)**

c. Check the validity of the following argument :
If the band could not play rock music or the refreshments were not delivered on time, then the New Year's party would have been cancelled and Alicia would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music. **(08 Marks)**

3 a. Write down the converse , inverse and contra positive of each of the following statements for which the set of all real numbers is the universe. Also indicate their truth values.
i) $\forall x, [(x > 3) \to (x^2 > 9)]$ ii) $\forall x, [\{(x^2 + 4x - 21) > 0\} \to \{(x > 3) \vee (x < -7)\}]$. **(06 Marks)**

b. Check the validity of the following arguments :
In triangle XYZ, there is no pair of angles of equal measure.
If a triangle has two sides of equal length , then it is isosceles.
If a triangle is isosceles , then it has two angles of equal measure.
Therefore triangle XYZ has no two sides of equal length. **(08 Marks)**

c. Let n be an integer, prove that , n is odd if and only if 7n + 8 is odd. **(06 Marks)**

4 a. Prove the following , for all $n \geq 1$ using the principle of Mathematical Induction :
$$\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$ **(06 Marks)**

b. Use Mathematical induction to prove that 5 divides $n^5 - n$, where n is a non – negative integer. **(06 Marks)**

c. For the Fibonacci sequence $F_0, F_1, F_2, \ldots$ ($F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$). Prove that

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

(08 Marks)

PART – B

5 a. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

(06 Marks)

i) Determine $f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$.

ii) Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(-6)$.

iii) What are $f^{-1}([-5, 5])$ and $f^{-1}([-6, 5])$?

b. Prove that function $f : A \to B$ is invertible if and only if F is one to one and On to. (08 Marks)

c. Using characteristic functions, prove that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$. For any sets A, B, C contained in a universal set U. (06 Marks)

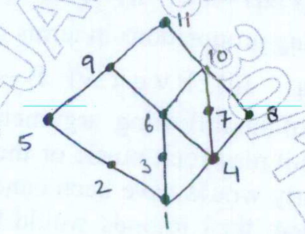
6 a. If $R = \{(x, y) / x > y\}$ is a relation defined on the set $A = \{1, 2, 3, 4\}$, write down the matrix and the digraph of R. Also list the in – degrees and out – degrees of all vertices. (06 Marks)

b. Define R on $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ as $(x, y) \in R$, if $x - y$ is a multiple of 5.

i) Show that R is an equivalence relation on A.

ii) Determine the equivalence classes and partition of A induced by R. (08 Marks)

c. For the Poset shown in the following Hasse diagram, find i) all upper bounds ii) all lower bounds iii) $L \cup B$ and GLB of the set A, where $A = \{6, 7, 10\}$. (06 Marks)



7 a. Prove that the cube roots of Unity form a group under usual multiplication. (06 Marks)

b. Prove that every subgroup of a cyclic group is cyclic. (06 Marks)

c. Let f be a homomorphism from a group G_1 to a group G_2. Prove the following:

i) If H_1 is a subgroup of G_1 and $H_2 = f(H_1)$, then H_2 is a subgroup of G_2.

ii) If f is an isomorphism from G_1 on to G_2, then f^{-1} is an isomorphism from G_2 on to G_1. (08 Marks)

8 a. Let C be a group code in Z_2^n. If $r \in Z_2^n$ is a received word and r is decoded as the code word. C^*, then prove that $d(C^*, r) \le d(C, r)$ for all $c \in C$. (06 Marks)

b. Find all integers K and m for which $(z, \oplus, \odot)$ is a ring under the binary operations $x \oplus y = x + y - K$, $x \odot y = x + y - mxy$. (06 Marks)

c. i) Prove that in Z_n, $[a]$ is a unit if and only if $\gcd(a, n) = 1$.

ii) Find all the units in Z_{12}. (08 Marks)

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